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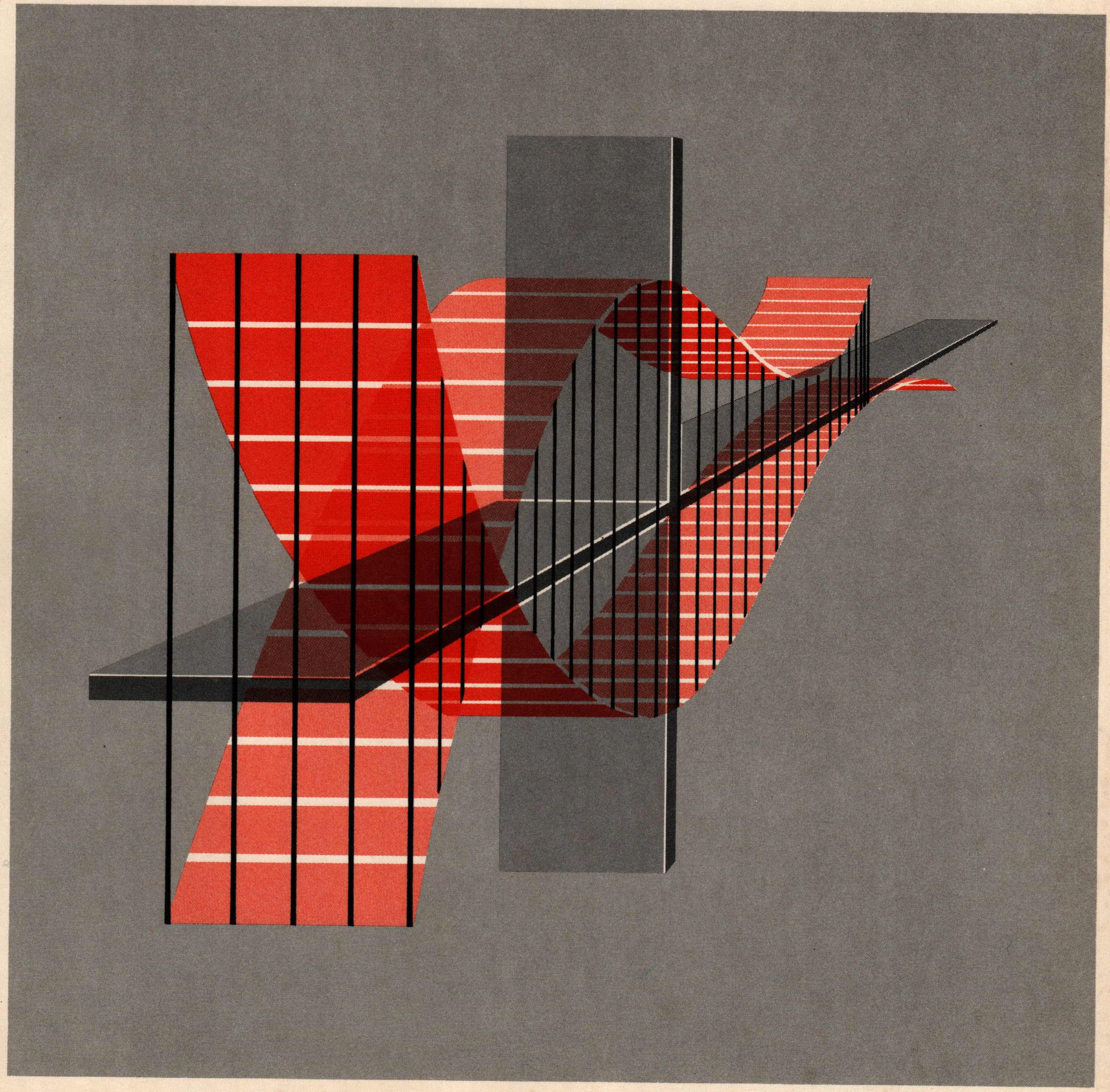

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COMPUTER - AUGMENTED CALCULUS TOPICS

The Curriculum material and programs contained in this booklet examine the basic concepts of calculus. Each topic is discussed in some detail in terms of its application to computer problem solving and a number of valuable problems and exercises accompany each item. The last section of the booklet deals with the graphing of functions and proposes several unique programs for determining functions, given a variety of related information.

This booklet is ideal as a teacher resource guide for computer applications in calculus, or it can be used as a student workbook to augment classroom text materials.

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computer-augmented calculus topics

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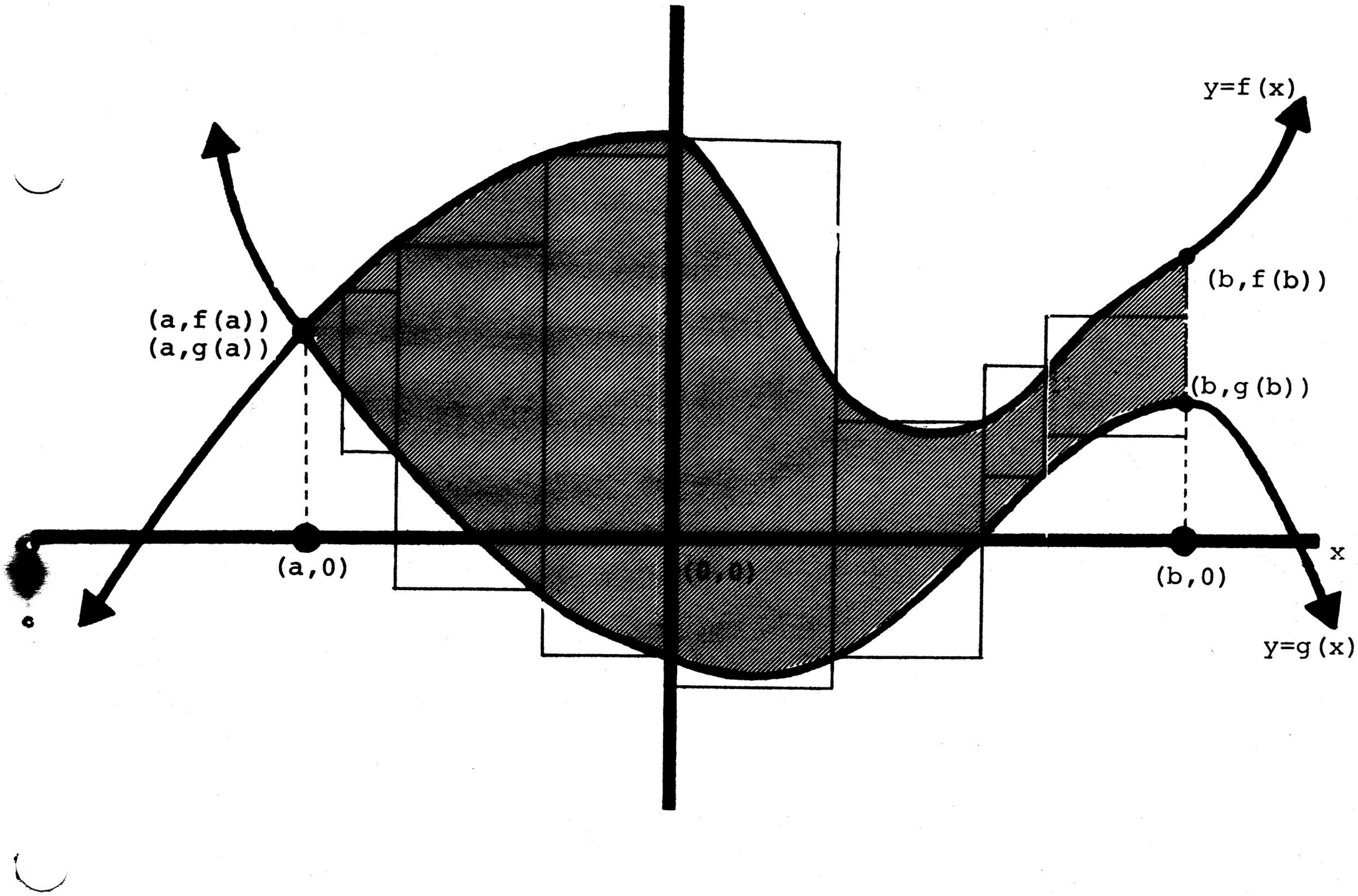


I. SOME HINTS ON THE LIMITATIONS OF COMPUTERS FOR CALCULATIONS

When you are solving some of the problems in this booklet you may encounter difficulties that appear to be caused by inconsistencies in mathematical methods but that are actually caused by the manner in which computers store and process numbers. Computers can not handle numbers which have too many significant digits or which are too large in absolute value. In NEWBASIC you are limited to numbers having ten significant digits that range in absolute value from 10^{-76} to 10^{+76}.[1] Problems also may arise when computers try to divide by numbers close to zero, subtract two numbers which are very close together, or add a very small decimal number to a very large whole number. One of the jobs which many mathematicians, computer scientists, and engineers work on is that of devising ingenious schemes for getting around these limitations of computers.

The following examples show a few of the kinds of mathematical problems that will be solved incorrectly by computers.

Example 1: The problem of finding 2^{793} can not be solved directly on a computer because 2^{793} is too large in absolute value to be calculated by an instruction such as `>10 LET Y=2.0↑793`. This statement will cause the computer to print out `X IN EXP(X) TOO LARGE LINE 10`.

Example 2: $8 + 10^{-13}$ is 8.0000000000001, but the instructions:

```
10 LET X=8.0
20 LET Y=1.0E-13
30 LET Z=X + Y
40 PRINT Z
50 END
```

will give the following printout for Z.

```
8
```

---

[1] EduSystem BASIC has an exponential limitation of ±38.

Example 3: The problem here is to find $\frac{x}{\sqrt{x+9}-3}$ for x=.000000001.

The program:

```
10 LET X=.000000001
20 LET Y=X/(SQRT(X+9.0)-3.0)
30 PRINT Y
40 END
```

gives 6.247225158 for the value of Y and this is not the correct solution to the problem.

However, we can use the computer to solve this problem if first we rationalize the denominator.

$$\frac{x}{\sqrt{x+9}-3} = \sqrt{x+9}+3$$

The program:

```
10 LET X=.000000001
20 LET Y=SQRT(X+9.0) +3.0
30 PRINT Y
40 END
```

gives the correct answer of 6 as output.

II. LIMITS OF SEQUENCES, LIMITS OF SUMS OF SEQUENCES, LIMITS OF A FUNCTION

The speed and accuracy of computers does make them particularly useful for exploring limiting processes in mathematics. For example the sequence defined by $f(n)=\sqrt[n]{n}$ (for $n=1,2,3,\cdots$) is made up of terms close to 1 for large values of n. The computation of a sufficient number of terms having large enough indices to enable us to be reasonably certain that $\lim_{n\to\infty} \sqrt[n]{n} = 1$ is quite tedious if we use pencil and and paper; however, the simple program:

```
10 INPUT N
20 LET F=N↑(1.0/N)
30 PRINT N, F
40 GOTO 10
```

permits us to compute enough terms of the sequence to discover that 1 is a good guess for the limit. The following table shows that this sequence converges to its limit rather slowly.

N	F	N	F	N	F	N	F
1	1.000	6	1.348	20	1.162	200	1.027
2	1.414	7	1.320	30	1.120	300	1.019
3	1.442	8	1.297	40	1.097	400	1.015
4	1.414	9	1.277	50	1.081	500	1.013
5	1.380	10	1.259	100	1.047	1000	1.007

Using a computer to "make a sequence approach its limit" can help to clarify our understanding of limits; however, we must exercise extreme caution in drawing conclusions. Since we can never look at all of the terms, any conclusion about the limit of the sequence must be considered at best an educated guess and is always suspect until we can find some way to prove our guess.

Problem 1: Write programs and use them to help you to make guesses about the limits of these sequences.

A. $\lim_{n\to\infty} \sqrt[n]{10}$

B. $\lim_{n\to\infty} \frac{2n+10}{n}$

C. $\lim_{n\to\infty} \frac{1}{2^n}$ (Be careful, for large values of n, 2^n is too large for the computer.)

D. $\lim_{n\to\infty} \frac{n^2}{2^n}$

E. $\lim_{n\to\infty} \frac{n+1}{n-1}$ (What happens if $n=1$?)

F. $\lim_{n\to\infty} \left(\frac{2}{3}\right)^n$

G. $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$ (This function has a limit, but for very large n the computer gives incorrect values for $\left(1+\frac{1}{n}\right)^n$.

Problem 2: For each real number p, $f(n)=\frac{1}{n^p}$ determines a sequence for $n=1,2,3,\cdots$. For some values of n the sequence converges and for others it deverges. Write a program that can be used to assist us in finding $\lim_{n\to\infty} \frac{1}{n^p}$ for any value of p. Use your program to find the values of p for which this sequence will converge.

Problem 3: Make up some interesting sequences and write programs to help you discover their limits.

The problem of finding the limit of the sum of the terms of a sequence can also be approached with the assistance of a computer. For instance the following program will sum the first n terms of the sequence 1, 1/2, 1/4, 1/8, $\cdots$ for any integral value of n.

```
10 LET S=1
20 INPUT N
30 FOR I=1 TO N
40 LET F=1.0/(2.0↑I)
50 LET S=S+F
60 PRINT I, S
70 NEXT I
80 END
```

For n=10 we have:

I	S	I	S
1	1.5	6	1.984375
2	1.75	7	1.9921875
3	1.875	8	1.99609375
4	1.9375	9	1.998046875
5	1.96875	10	1.999023437

From this table it appears that the limit may be 2.

Problem 4: Write programs and use them to estimate the limits of the sums of these sequences:

(A) $1, 1/3, 1/9, 1/27, \cdots$

(B) $(1/2)^2, (1/3)^2, (1/4)^2, \cdots$

(C) $\frac{1}{0!}, \frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, \cdots$

(D) $1, -1/2, 1/3, -1/4, 1/5, \cdots$

(E) $1, 1/2, 1/3, 1/4, 1/5, \cdots$

We can build upon our procedures for exploring limits of sequences and limits of sums of sequences to study the limit of a function defined on any subset of the real numbers.

For example, consider $\lim_{x\to 1} \frac{x^3+x^2-x-1}{x-1}$. Since x can have any real value except 1, we must decide upon a way to let x approach 1 in order to obtain useful data about this limit. One of the many possibilities is shown next.

This program with N=5

```
10 INPUT N
15 PRINT
20 FOR I=1 TO N
30 LET D=10↑(-I)
40 LET X1=1-D
50 LET X2=1+D
60 LET Y1=(X1↑3+X1↑2-X1-1)/(X1-1)
70 LET Y2=(X2↑3+X2↑2-X2-1)/(X2-1)
```

```
 80 PRINT X1,Y1
 90 PRINT X2,Y2
100 NEXT I
110 END
```

gives this output.

```
0.9            3.61
1.1            4.41
0.99           3.960100001
1.01           4.040099998
0.999          3.99600101
1.001          4.004000991
0.9999         3.999600113
1.0001         4.000399887
0.99999        3.99996071
1.00001        4.00003929
```

The output indicates (but certainly doesn't prove) that the limit may be 4.

You can see that the programming skills needed to write programs for exploring limits of sequences and functions are minimal and even people with little experience on the terminal can soon be exploring some rather sophisticated mathematics concepts.

Problem 5: Write programs and use them to make an educated guess about the following limits:

(A) $\lim_{x \to 0} \left(\frac{1}{1+x}\right)^x$

(B) $\lim_{x \to 0} \frac{\sin(x)}{x}$

(C) $\lim_{x \to 1} \frac{x^2-1}{x-1}$

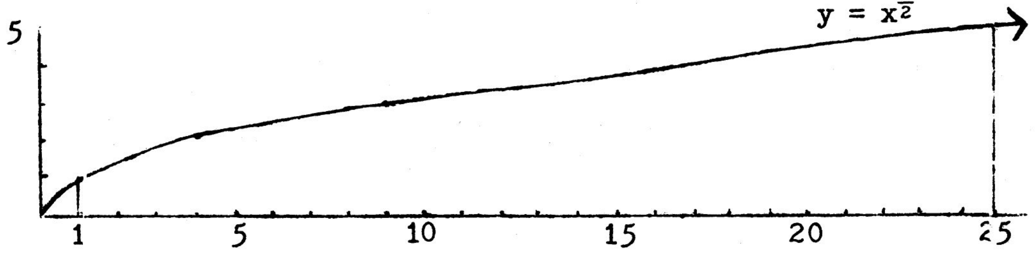
(D) $\lim_{x \to 0} \frac{x}{\sqrt{x+4} - 2}$ (See Example 3)

(E) $\lim_{x \to 0} (1+x)^{1/x}$ (You may run into a problem here.)

(F) Compare Problem 5, part E to Problem 1, part G.

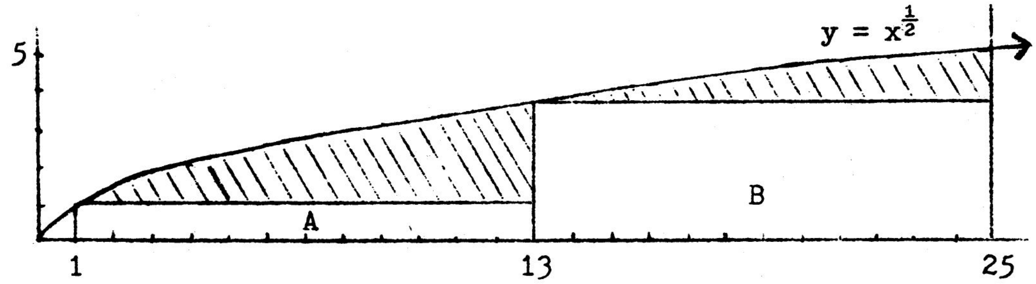
III. APPROXIMATING AREAS

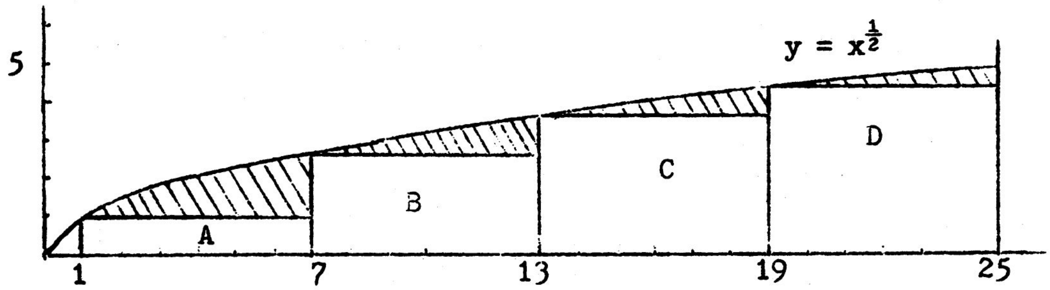
Our problem now is that of finding approximations to **areas** bounded by the graph of a function and an interval of the x-axis. For example suppose we want to find the area bounded by the graph of $y=x^{\frac{1}{2}}$ and the interval [1, 25] of the x-axis. This area is illustrated in the following figure.

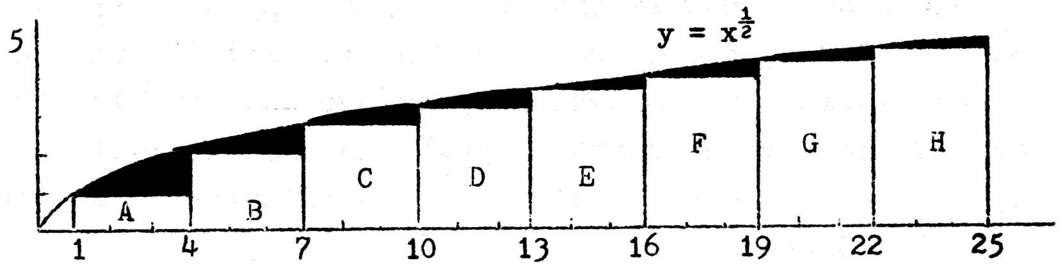


The first approach we will use to approximate this area is that of constructing rectangles, the sum of whose areas is an approximation to the area in question.

The next set of diagrams illustrates ways of building rectangles to get increasingly more accurate approximations.





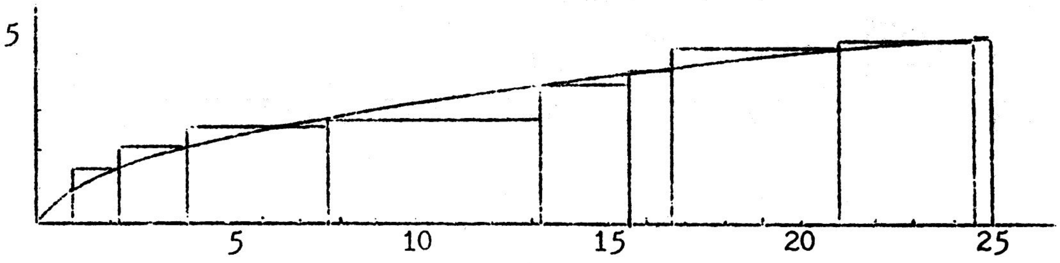


With two rectangles the approximation is $12(1)^{\frac{1}{2}} + 12(13)^{\frac{1}{2}} = 53.27$. With four rectangles it is $6(1)^{\frac{1}{2}} + 6(7)^{\frac{1}{2}} + 6(13)^{\frac{1}{2}} + 6(19)^{\frac{1}{2}} = 69.66$. With eight rectangles our approximation is 76.39.

You can observe from the shaded areas in the figures that the difference between the area and our approximation (the error) decreases as we increase the number of rectangles. The procedure of doubling the number of rectangles was continued to 8192 rectangles to obtain the following table.

Number of rectangles	Approximation to the area	Number of rectangles	Approximation to the area
2	55.266	128	82.291
4	69.661	256	82.479
8	76.388	512	82.573
16	79.594	1024	82.620
32	81.148	2048	82.643
64	81.912	4096	82.655
		8192	82.661

We could have constructed some of the rectangles to extend above the graph and the widths of their bases could have been varied as shown below.



The next program /AREA/ is an example of the "rectangle method" (Eulers method) for obtaining a set of approximations to the area bounded by the graph of any specified polynomial of degree three or less for any interval of the x-axis. When you use the program you are free to select the polynomial and the interval. After the program has calculated an approximation to the area bounded by $ax^3 + bx^2 + cx + d$ over [r,s] four options are available to you:

1. Use more rectangles to get a better approximation.
2. Approximate an area bounded by the same function over a different interval.
3. Approximate the area bounded by a different function.
4. Quit and do something else.

When using the program, after you get the message NOW TYPE EITHER 34, 44, 54, OR 1000

1. Type 54 for option 1.
2. Type 44 for option 2.
3. Type 34 for option 3.
4. Type 1000 for option 4.

A sample run together with the program listing is shown on the next two pages.

A sample run of the program /AREA/.

```
RUN

THIS PROGRAM COMPUTES APPROXIMATIONS TO THE AREA
BOUNDED BY THE X-AXIS AND THE GRAPH OF ANY POLYNOMIAL
FUNCTION OF DEGREE THREE OR LESS OVER AN INTERVAL OF
THE X-AXIS.  THE METHOD USED TO APPROXIMATE THE AREA
IS TO BUILD RECTANGLES WHOSE SUM WILL BE AN APPROXIMATION
TO THE AREA WE ARE TRYING TO FIND.
THE POLYNOMIAL FUNCTION IS OF THE FORM

A*X↑3+B*X↑2+C*X+D

YOU CAN GIVE ANY VALUES (INCLUDING 0) TO A,B,C, AND D.
O.K., ASSIGN NUMERICAL VALUES TO A,B,C, AND D.
A= ? 0
B= ? 1
C= ? 0
D= ? 0
```

```
SELECT AN INTERVAL (R,S) ON THE X-AXIS
BY ASSIGNING VALUES TO R AND S.
R= ? 0
S= ? 2
HOW MANY RECTANGLES DO YOU WANT TO BUILD ON (R,S)?
N= ? 4

THE INTERVAL IS ( 0 , 2 ).
THE NUMBER OF RECTANGLES IS  4 .
THE FUNCTION IS
 0 X↑3+  1 X↑2+  0 X+  0 .

THE APPROXIMATION TO THE AREA IS ***** 1.75 *****


NOW TYPE EITHER 34,44,54, OR 1000.
? 54
HOW MANY RECTANGLES DO YOU WANT TO BUILD ON (R,S)?
N= ? 8

THE INTERVAL IS ( 0 , 2 ).
THE NUMBER OF RECTANGLES IS  8 .
THE FUNCTION IS
 0 X↑3+  1 X↑2+  0 X+  0 .

THE APPROXIMATION TO THE AREA IS ***** 2.1875 *****


NOW TYPE EITHER 34,44,54, OR 1000.
? 54
HOW MANY RECTANGLES DO YOU WANT TO BUILD ON (R,S)?
N= ? 64

THE INTERVAL IS ( 0 , 2 ).
THE NUMBER OF RECTANGLES IS  64 .
THE FUNCTION IS
 0 X↑3+  1 X↑2+  0 X+  0 .

THE APPROXIMATION TO THE AREA IS ***** 2.60449 *****


NOW TYPE EITHER 34,44,54, OR 1000.
? 1000
```

A listing of the program /AREA/.

```
10 PRINT "THIS PROGRAM COMPUTES APPROXIMATIONS TO THE AREA"
12 PRINT"BOUNDED BY THE X-AXIS AND THE GRAPH OF ANY POLYNOMIAL"
14 PRINT"FUNCTION OF DEGREE THREE OR LESS OVER AN INTERVAL OF"
16 PRINT"THE X-AXIS.  THE METHOD USED TO APPROXIMATE THE AREA"
18 PRINT "IS TO BUILD RECTANGLES WHOSE SUM WILL BE AN APPROXIMATION"
20 PRINT"TO THE AREA WE ARE TRYING TO FIND."
24 PRINT"THE POLYNOMIAL FUNCTION IS OF THE FORM"
26 PRINT
```

```
28 PRINT"A*X↑3+B*X↑2+C*X+D"
30 PRINT
32 PRINT"YOU CAN GIVE ANY VALUES (INCLUDING 0) TO A,B,C, AND D."
34 PRINT"O.K., ASSIGN NUMERICAL VALUES TO A,B,C, AND D."
36 PRINT "A= ";
37 INPUT A
38 PRINT"B= ";
39 INPUT B
40 PRINT"C= ";
41 INPUT C
42 PRINT"D= ";
43 INPUT D
44 PRINT "SELECT AN INTERVAL (R,S) ON THE X-AXIS"
46 PRINT "BY ASSIGNING VALUES TO R AND S."
48 PRINT "R= ";
49 INPUT R
50 PRINT "S= ";
51 INPUT S
52 IF R>=S GOTO 500
54 PRINT "HOW MANY RECTANGLES DO YOU WANT TO BUILD ON (R,S)?"
56 PRINT "N= ";
57 INPUT N
58 PRINT
60 LET P=INT(N)
61 IF N>500 GOTO 600
62 IF N<1 GOTO 600
63 IF N<>P GOTO 600
64 LET T=0
66 LET W=(S-R)/N
70 FOR X=R TO (S-W) STEP W
72 LET T=T+ ABS(A*X↑3+B*X↑2+C*X+D)
74 NEXT X
76 LET T=T*W
78 PRINT "THE INTERVAL IS (";R;",";S;")."
80 PRINT "THE NUMBER OF RECTANGLES IS ";N;"."
82 PRINT "THE FUNCTION IS"
84 PRINT A;"X↑3+ ";B;"X↑2+ ";C;"X+ ";D;"."
86 PRINT
88 PRINT "THE APPROXIMATION TO THE AREA IS *****";T;"*****"
89 PRINT
90 PRINT
104 PRINT "NOW TYPE EITHER 34,44,54, OR 1000."
106 INPUT K
110 IF K=34 GOTO 34
111 IF K=44 GOTO 44
112 IF K=54 GOTO 54
113 IF K=1000 GOTO 1000
114 GOTO 700
500 PRINT"R MUST BE  LESS THAN S. TRY AGAIN>"
505 GOTO 44
600 PRINT"N MUST BE A POSITIVE WHOLE NUMBER"
605 PRINT "LESS THAN 500. TRY AGAIN."
610 GOTO 54
700 PRINT "YOU GOOFED. TAKE IT FROM THE TOP."
710 GOTO 104
1000 END
```

Problem 6: Use the program /AREA/ to find good approximations to the following areas:

A. x^2 over [0, 4]
B. x^2 over [-4, 4]
C. x^3 over [-2, 0]
D. x^3 over [0, 2]
E. x^3 over [-2, 2]
F. $2x^3-3x^2+x-1$ over [-4, 3]

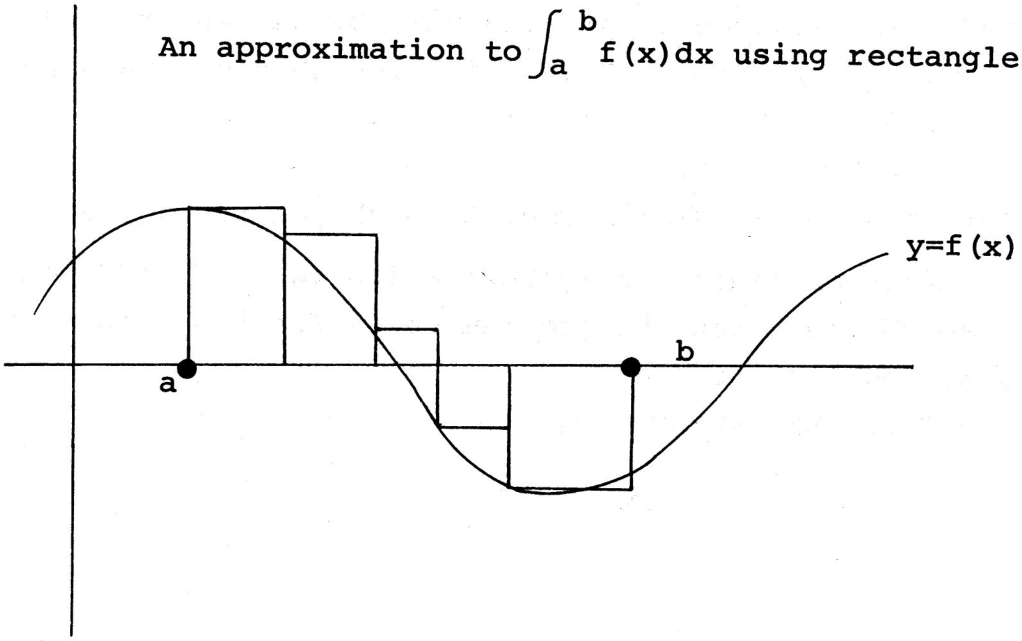
Problem 7: Write a program and use it to approximate the area bounded by sin(x) over the intervals.

A. [-π, 0]
B. [0, π]
C. [-π, π]

IV. NUMERICAL INTEGRATION

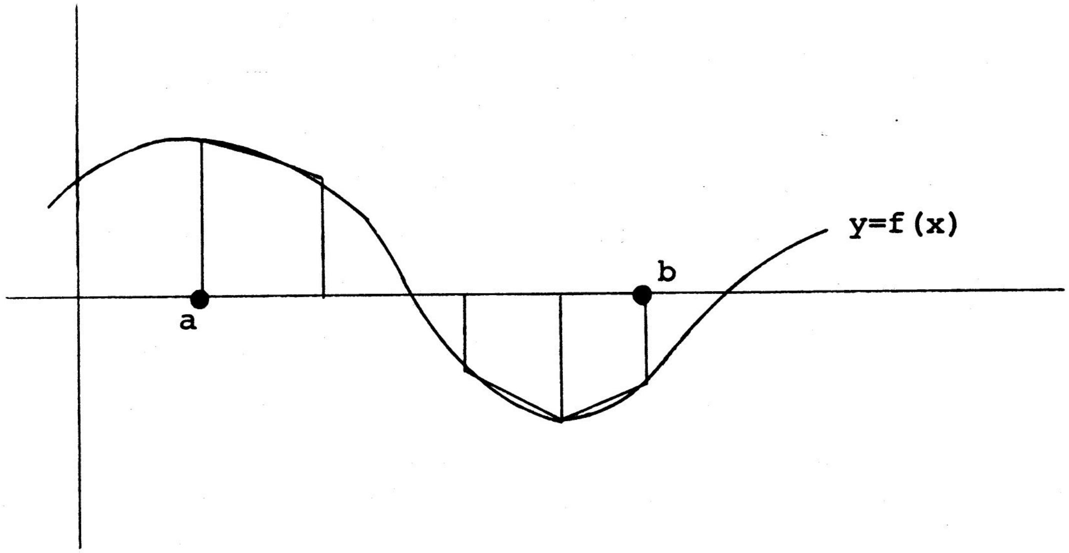
The method of constructing rectangles to approximate areas is one of several strategies which can be used in numerical integration. Remember that we can define the integral $\int_a^b f(x)dx$ to be $\lim_{\Delta x_i \to 0} \sum_{i=1}^{n} (\Delta x_i) f(x_i)$, where the Δx_i's are subintervals of the x-axis which partition the interval [a, b] and the $f(x_i)$'s are respective points in these intervals. To obtain a set of approximations to $\int_a^b f(x)dx$, we need to compute $\sum_{i=1}^{n} (\Delta x_i) f(x_i)$ where n, the number of subintervals into which [a, b] is divided, is increased for each approximation and the Δx_i's are decreased. In effect we are constructing rectangles of base Δx_i with height $f(x_i)$, for i=1, 2, 3, 4,···, and summing the areas of all the rectangles. Since some of the $f(x_i)$'s could be negative, which occurs over subintervals where the graph of f(x) dips below the x-axis, some of our areas $(\Delta x_i) f(x_i)$ will be negative.

The next program /INTEGRATE/ is an illustration of the trapezoid method for approximating the definite integral of some function f(x). In this method we use approximating trapezoids rather than approximating rectangles as is shown in the diagram.



An approximation to $\int_a^b f(x)dx$ using rectangles.

An approximation to $\int_a^b f(x)dx$ using trapezoids.



After you have loaded the program /INTEGRATE/ do <u>not</u> type RUN. You must alter the program by supplying it with the function whose definite integral you want to approximate. This function must be entered as line 100 of the program. For example if you want to integrate the function $y=x^2 + 2x - 1$ over some interval, you must type

```
100 LET Y=X↑2 + 2.0*X -1.0.
```

Now the program is complete and you can use it by typing RUN. The dependent variable must be called Y and the independent variable X; so line 100 should always have the form

```
100 LET Y=F(X)
```

where F(X) is the function of X which you are going to study.

After obtaining an approximation to $\int_a^b f(x)dx$ you have three choices when the program calls for the change code by typing the message

```
TYPE THE CHANGE CODE.
```

Choice 1: Type 30 to get a better approximation by increasing the number of trapezoids.
Choice 2: Type 18 to integrate the same function over a different interval.
Choice 3: Type 300 to stop the program.

If you type 300 you can either stop or try the program on a different function Y=G(X) by typing

```
100 LET Y=G(X)
RUN
```

Problem 8: Use /INTEGRATE/ to obtain a set of approximations to $\int_0^2 e^x dx$. Make a guess as to the value of this integral.

Problem 9: Use integrate to approximate:

A. $\int_1^2 \log_{10}(x)dx$

B. $\int_2^4 \log_{10}(x)dx$

C. $\int_1^4 \log_{10}(x)dx$

Problem 10: Use /INTEGRATE/ to approximate $\int_0^1 \log_2(x)dx$. Some ingenuity is required here because $\log_2(0)$ is undefined.

Problem 11: Sketch a graph of $y=x^3-9x$. Use /INTEGRATE/ to approximate the following definite integrals.

A. $\int_{-4}^3 (x^3-9x)dx$

B. $\int_{-3}^0 (x^3-9x)dx$

C. $\int_0^3 (x^3-9x)dx$

D. $\int_{-4}^{-3} (x^3-9x)dx$

Interpret the results of A, B, C, D relative to your graph.

A sample run of the program /INTEGRATE/.

```
100 LET Y=X↑2-2.0*X
RUN

THIS PROGRAM COMPUTES APPROXIMATIONS TO THE DEFINITE
INTEGRAL OF THE FUNCTION WHICH YOU SUPPLIED ON LINE 100
OVER AN INTERVAL (A,B).

NOW TYPE A VALUE FOR A.
?1
TYPE THE VALUE OF B.
?3
HOW MANY SUBINTERVALS DO YOU WANT (A,B) DIVIDED INTO?
?16


THE INTERVAL IS ( 1 , 3 ).
THE NUMBER OF APPROXIMATING TRAPEZOIDS IS 16

THE APPROXIMATION IS   ****** .671875 ******

TYPE THE CHANGE CODE.
?30
HOW MANY SUBINTERVALS DO YOU WANT (A,B) DIVIDED INTO?
?100


THE INTERVAL IS ( 1 , 3 ).
THE NUMBER OF APPROXIMATING TRAPEZOIDS IS 100

THE APPROXIMATION IS   ****** .696796 ******

TYPE THE CHANGE CODE.
?30
HOW MANY SUBINTERVALS DO YOU WANT (A,B) DIVIDED INTO?
?400


THE INTERVAL IS ( 1 , 3 ).
THE NUMBER OF APPROXIMATING TRAPEZOIDS IS 400

THE APPROXIMATION IS   ****** .673972 ******

TYPE THE CHANGE CODE.
?500
THE CHANGE CODE MUST BE 18,30, OR 300.
TYPE THE CHANGE CODE.
?30
HOW MANY SUBINTERVALS DO YOU WANT (A,B) DIVIDED INTO?
?500


THE INTERVAL IS ( 1 , 3 ).
THE NUMBER OF APPROXIMATING TRAPEZOIDS IS 500

THE APPROXIMATION IS   ****** .672516 ******

TYPE THE CHANGE CODE.
?300
```

A listing of the program /INTEGRATE/.

```
10 PRINT "THIS PROGRAM COMPUTES APPROXIMATIONS TO THE DEFINITE"
12 PRINT "INTEGRAL OF THE FUNCTION WHICH YOU SUPPLIED ON LINE 100"
14 PRINT "OVER AN INTERVAL (A,B)."
16 PRINT
18 PRINT "NOW TYPE A VALUE FOR A."
19 INPUT A
20 PRINT "TYPE THE VALUE OF B."
21 INPUT B
25 IF A>=B GO TO 200
30 PRINT "HOW MANY SUBINTERVALS DO YOU WANT (A,B) DIVIDED INTO?"
35 INPUT N
40 LET N=INT(ABS(N))
45 LET S=0
50 LET W=(B-A)/N
55 FOR X=A TO B STEP W
100 LET Y=X↑2-2.0*X
105 IF X=B GOTO 125
110 IF X=A GOTO 125
115 LET S=S+2*Y
120 GOTO 130
125 LET S=S+Y
130 NEXT X
135 LET T=((B-A)/(2*N))*S
140 PRINT
142 PRINT
144 PRINT "THE INTERVAL IS (";A;",";B;")."
146 PRINT "THE NUMBER OF APPROXIMATING TRAPEZOIDS IS";N
148 PRINT
150 PRINT "THE APPROXIMATION IS    ******";T;"******"
152 PRINT
155 PRINT "TYPE THE CHANGE CODE."
156 INPUT C
160 IF C=18 GOTO 18
165 IF C=30 GOTO 30
170 IF C=300 GOTO 300
175 GOTO 250
200 PRINT "A MUST BE LESS THAN B. TRY AGAIN."
205 GOTO 18
250 PRINT "THE CHANGE CODE MUST BE 18,30, OR 300."
255 GOTO 155
300 END
```

V. APPROXIMATING VALUES OF DERIVATIVES

The derivative of f(x) at x=a can be defined as

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$ Since a is a constant, $\frac{f(a+h)-f(a)}{h}$ is a function of h; consequently any method developed for approximating the limit of a function can be used to approximate this limit.

To approximate the derivative of $y=x^2$ for x=3 directly from definition, we need to find values of $\frac{(3+h)^2-(3)^2}{h}$ for values of h which are close to 0. The program shown below produces the following set of approximations to $\lim_{h\to 0} \frac{(3+h)^2-(3)^2}{h}$, where N=5. Notice that in this program h is approaching 0 from the positive side.

```
10 INPUT N
20 FOR I=1 TO N
30 LET H=1/(10↑N)
40 LET D=(((3+H)*2)-(3*2))/H
50 PRINT H, D
60 NEXT I
70 END
```

h	$\frac{(3+h)^2-(3)^2}{h}$
.1	6.099999999
.01	6.009999994
.001	6.000999885
.0001	6.000099238
.00001	5.999999121

<u>Problem 12</u>: Write a program which will compute values of $\frac{(3+h)^2-(3)^2}{h}$ for negative values of h. Use your program to estimate $\lim_{h\to 0} \frac{(3+h)^2-(3)^2}{h}$ as h approaches 0 from the negative side. What is the derivative of x^2 when x = 3?

The problem of finding $\frac{(3+h)^2-(3)^2}{h}$ for values of h which are close to 0 illustrates one of the difficulties that can occur when we use a computer to perform our calculations. Suppose h=.00000000001, then the computer must find $\frac{(3+.00000000001)^2-3^2}{.00000000001}$. However, 3+.00000000001 = 3.00000000001, which has 12 significant digits, and our computer can't handle this many significant digits, consequently the computer will take 3.00000000001 to be 3.0000000000 and compute $\frac{(3.0000000000)^2-(3.0000000000)^2}{1.0E-11}$ which is 0. The correct answer is 6.00000000001, and in fact we are going to get erroneous data when $|h|<.0001$. We can avoid this problem by simplifying $\frac{(3+h)^2-(3)^2}{h}$ before we write our computer program.

$$\frac{(3+h)^2-(3)^2}{h} = \frac{9+6h+h^2-9}{h} = 6+h$$

So, $\lim_{h\to 0} \frac{(3+h)^2-(3)^2}{h} = \lim_{h\to 0} (6+h)$, and for h close to 0, 6+h is close to 6.

If we don't make this simplification the computer gives us incorrect information. If we make the simplification we don't really need a computer to show us that $\lim_{h\to 0} (6+h) = 6$. In this problem the computer really isn't of much help.

As a general rule when using the computer to find $\frac{f(x+h)-f(x)}{h}$ for various values of x, you should avoid letting h be closer than .00001 to 0 as you attempt to estimate $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Problem 13: Write a program which can be used to find estimates for the derivative of $\log_{10} x$ for any value of x which the user of the program chooses. You will need to consider finding $\lim_{h\to 0} \frac{\log_{10}(x+h)-\log_{10}(x)}{h}$.

Remember that $\log_{10}x$ is only defined for positive values of x. Test your program by estimating the derivatives for x=10, 100, 1, 2, and 5.

The next program called /DERIVSIN/ is one which can be used to find the derivative function of sin(x). Our strategy will be to find approximations to $\lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$ for many values of x, obtaining a set of pairs of numbers $\{(x, \text{approximation to } \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h})\}$ for these values of x. We will then plot all of these points on a rectangular coordinate system, sketch the graph which they form, and try to guess the function which has this graph. For each value of x that is entered into the program, it will give as output x, sin(x) and values of h and $\frac{\sin(x+h)-\sin(x)}{h}$ for h=±.1, ±.01, ±.001, and ±.0001.

Problem 14: Use the program /DERIVSIN/ to find the derivative function of sin(x). Select values for x until you have enough approximations to the derivative of sin(x) to sketch reasonably good graphs of both sin(x) and D_x sin(x). What is your guess as to the derivative function of sin(x)?

Problem 15: Write a program similar to /DERIVSIN/ which for any value of x will give as output

$$x, \quad e^x$$

$$\text{and } h, \quad \frac{e^{x+h}-e^x}{h}$$

for h=±.1, ±.01, ±.001, and ±.0001.

Use this program to sketch graphs of $y=e^x$ and the derivative function of e^x.

Problem 16: Write a program which for any value of x will give as output x, cos(x) and h, $\frac{\cos(x+h)-\cos(x)}{h}$

for h=±.1, ±.01, ±.001, and ±.0001.

Use this program to discover the derivative function of cos(x).

A sample run of the program /DERIVSIN/.

```
PROGRAM TO ESTIMATE THE DERIVATIVE OF SIN(X) FOR
ANY VALUE OF X.

WHAT IS X?
?-1


X = SIN(X) = -.841471

    H           (SIN(X+H)-SIN(X))/H

 1E-1           .581437
-1E-1           .497367
 1E-2           .544476
-1E-2           .53612
 1E-3           .540376
-1E-3           .539899
 1E-4           .537634
-1E-4           .542402

WHAT IS X?
?0


X = SIN(X) =  0

    H           (SIN(X+H)-SIN(X))/H

 1E-1           .998334
-1E-1           .998334
 1E-2           .999983
-1E-2           .999983
 1E-3           1
-1E-3           1
 1E-4           1
-1E-4           1

WHAT IS X?
?1


X = SIN(X) =  .841471

    H           (SIN(X+H)-SIN(X))/H

 1E-1           .497367
-1E-1           .581437
 1E-2           .53612
-1E-2           .544476
 1E-3           .539899
-1E-3           .540376
 1E-4           .542402
-1E-4           .537634

WHAT IS X?
?
```

A listing of the program /DERIVSIN/.

```
10 PRINT "PROGRAM TO ESTIMATE THE DERIVATIVE OF SIN(X) FOR"
12 PRINT "ANY VALUE OF X."
14 PRINT
16 PRINT"WHAT IS X?"
17 INPUT X
18 PRINT
20 PRINT
30 LET Z=SIN(X)
40 PRINT "X = ";"SIN(X) = ";Z
42 PRINT
50 PRINT "    H           (SIN(X+H)-SIN(X))/H"
52 PRINT
60 FOR N=1 TO 4
70 LET H1=10↑(-N)
80 LET H2=-H1
90 LET Y1=(SIN(X+H1)-Z)/H1
100 LET Y2=(SIN(X+H2)-Z)/H2
110 PRINT H1,Y1
120 PRINT H2,Y2
130 NEXT N
140 GOTO 14
150 END
```

The following program is a generalization of the program /DERIVSIN/ which can be used to explore the derivative function of sin(x). When using this program, you have the option of selecting any function whose derivative you want to study. To use this program called /DERIVTIV/, first load the function into your workspace. Don't try to run this program until you alter it by providing the function whose derivative you are going to study. You can do this by typing

```
100 LET Y=F(X)
```

where F(X) is any function you want. For example to find approximations to values of the derivative function of $y=\sqrt{x+4}$ you must type

```
100 LET Y=SQRT(X+4.0)
```

Of course you must remember that x must be greater than or equal to -4 or SQRT(X+4.0) will be imaginary.

When you finish using the program to explore your function, hit the ESCAPE key. Now you can either LOGOUT or work on another function y=G(X) by typing

```
100 LET Y=G(X)
RUN
```

where G(X) is a specified function of X.

The following pages contain a set of sample output from /DERIVTIV/ and a listing of the program.

Problem 17: Use /DERIVTIV/ to obtain enough data to plot an accurate graph of $y=\frac{1}{x}$ and the derivative function of $y=\frac{1}{x}$. Plot these graphs and try to guess the function which is the derivative of $y=\frac{1}{x}$.

Problem 18: Use /DERIVTIV/ to assist in plotting the graphs of $y=|x|$ and its derivative function. Try to guess the derivative function of $y=|x|$.

Problem 19: Use /DERIVTIV/ to assist in plotting the graphs of y=sin(2x) and the derivative function of y=sin(2x). Can you guess what this derivative function is?

A sample run of the program /DERIVTIV/.

```
    100 LET Y=EXP(X)
```

```
RUN

THIS PROGRAM WILL COMPUTE  A SET OF APPROXIMATIONS TO THE
DERIVATIVE OF ANY FUNCTION F(X) WHICH YOU GIVE IT FOR
ANY VALUE YOU ASSIGN TO THE INDEPENDENT VARIABLE X,PROVIDED
THE FUNCTION HAS A DERIVATIVE FOR THAT X.

FOR EACH X YOU WILL GET AS OUTPUT THE VALUES OF X AND
F(X), AS WELL AS A SET OF APPROXIMATIONS TO F'(X) FOR
H = .1,-.1,.01,-.01,.001,-.001,.0001,-.0001.


WHAT IS X?
?0

     H        APPROXIMATION TO F'(X)
 1E-1            1.0517
-1E-1            .951624

 1E-2            1.005
-1E-2            .994993

 1E-3            1.0004
-1E-3            .999451

 1E-4            .998974
-1E-4            .998974


X =  0  Y =  1


WHAT IS X?
?1

     H        APPROXIMATION TO F'(X)
 1E-1            2.85884
-1E-1            2.58678

 1E-2            2.73184
-1E-2            2.70471

 1E-3            2.71892
-1E-3            2.71702

 1E-4            2.72274
-1E-4            2.7132


X =  1  Y =  2.71828


WHAT IS X?
?
```

A listing of the program /DERIVTIV/.

```
4 PRINT"THIS PROGRAM WILL COMPUTE  A SET OF APPROXIMATIONS TO THE"
6 PRINT "DERIVATIVE OF ANY FUNCTION F(X) WHICH YOU GIVE IT FOR"
8 PRINT "ANY VALUE YOU ASSIGN TO THE INDEPENDENT VARIABLE X, PROVIDED"
10 PRINT"THE FUNCTION HAS A DERIVATIVE FOR THAT X."
14 PRINT
16 PRINT"FOR EACH X YOU WILL GET AS OUTPUT THE VALUES OF X AND"
18 PRINT"F(X), AS WELL AS A SET OF APPROXIMATIONS TO F'(X) FOR"
20 PRINT "H = .1,-.1,.01,-.01,.001,-.001,.0001,-.0001."
24 PRINT
25 PRINT
26 PRINT
28 PRINT"WHAT IS X?"
29 INPUT X
30 PRINT
32 PRINT "     H        APPROXIMATION TO F'(X)"
35 FOR N=1 TO 4
40 LET H=10↑(-N)
45 LET C=0
47 LET I=0
50 LET A=X
55 LET X=X+H
100 LET Y=EXP(X)
110 IF I=1 GOTO 200
120 LET I=I+1
130 LET D1=Y
140 LET X=A
150 GOTO 100
200 LET D2=Y
210 LET D=(D1-D2)/H
220 PRINT H,D
230 LET C=C+1
240 IF C=2 GOTO 270
250 LET H=(-H)
260 GOTO 47
270 PRINT
300 NEXT N
320 PRINT
330 PRINT
340 PRINT "X = ";A; " Y = ";D2
350 GOTO 25
360 END
```

VI. THE FUNDAMENTAL THEOREM OF INTEGRAL CALCULUS

A modification of /INTEGRATE/ can be used to "numerically verify" The Fundamental Theorem of Integral Calculus; i.e., $\int_a^b f(x)dx=F(b)-F(a)$ where F(x) is a primitive of f(x).

For example,

$$\int_0^{\pi/2} \sin(x)dx=\left[-\cos(\pi/2)\right]-\left[-\cos(0)\right]$$

can be verified by calculating a sequence of approximations to $\int_0^{\pi/2} \sin(x)dx$ and comparing the suspected limit of the sequence to $\left[-\cos(\pi/2)\right]-\left[-\cos(0)\right]=1$.

The following program, /FUNDTHEO/, uses the trapezoid method to compute approximations to $\int_a^b f(x)dx$ for a given function f(x) over a specified interval [a, b] for n trapezoids, and compares the approximation to g(b)-g(a) where g(x) is a primitive of f(x).

After you have loaded the program, modify lines 100 and 200 as follows:

Line 100 should contain the function to be integrated, Y=F(X) in the form:

```
100 LET Y=F(X)
```

Line 200 should contain any primitive Z=G(X) of Y=F(X) in the form:

```
200 LET Z=G(X)
```

For example, if the integral is $\int_a^b \cos(x)dx$, then line 100 should be:

```
100 LET Y=COS(X)
```

and line 200 could be:

```
200 LET Z=SIN(X)+1
```

since SIN(X)+1 is a primitive of COS(X).

After you have supplied lines 100 and 200 you can execute the program by typing

```
RUN
```

After the program has printed an approximation to $\int_a^b f(x)dx$ and G(B)-G(A) it will print

TYPE THE CODE

To use a different number of approximating trapezoids on the interval [a, b] type 30; to get a new interval by changing the values of a and b, type 20; to stop the program type 300.

Problem 20: Use /FUNDTHEO/ to approximate $\int_0^2 -x^2 dx$ and to verify that $\int_0^2 -x^2 dx = \left[-\frac{x^3}{3} + 2 \right]_0^2$.

Problem 21: Use /FUNDTHEO/ to approximate $\int_0^{\pi/2} \cos^2(x)dx$ and to verify that $\int_0^{\pi/2} \cos^2(x)dx = \left[\frac{x}{2} + \frac{\sin(x)\cos(x)}{2} - 1 \right]_0^{\pi 2}$

Problem 22: Use /FUNDTHEO/ to verify $\int_0^1 \frac{dx}{x^2+1} = \left[\arctan(x) \right]_0^1$

Problem 23: Use /FUNDTHEO/ to verify

A. $\int_2^4 e^{3x}dx = \left[\frac{e^{3x}}{3} - 6 \right]_2^4$

B. $\int_2^3 \log_e(x)dx = \left[x(\log_e x) - x \right]_2^3$

C. $\int_{\frac{1}{2}}^1 \sec^2(x)dx = \left[\tan(x) - 2 \right]_{\frac{1}{2}}^1$

D. $\int_{-1}^1 \sqrt{1+\cos(x)}\, dx = \left[2\sqrt{2}\,(\sin\frac{x}{2}) \right]_{-1}^1$

A sample run of the program /FUNDTHEO/.

```
RUN
THIS PROGRAM COMPUTES APPROXIMATIONS TO THE DEFINITE
INTEGRAL OF THE FUNCTION Y=F(X) WHICH YOU SUPPLIED ON
LINE 100, OVER AN INTERVAL (A,B); AND COMPARES THE
APPROXIMATION TO G(B)-G(A), WHERE Z=G(X) IS A
PRIMITIVE OF F(X), WHICH YOU SUPPLIED IN LINE 200.

WHAT IS A? -3.141592
WHAT IS B? 0
HOW MANY SUBINTERVALS ON (A,B)? 200

INTERVAL (-3.141592 , 0 ). 200 TRAPEZOIDS

APPROXIMATION TO INTEGRAL IS *****-1.999959 *****

G(B) - G(A) IS -1    -    1  = -2


TYPE THE CODE? 20

WHAT IS A? 0
WHAT IS B? 3.141592
HOW MANY SUBINTERVALS ON (A,B)? 200

INTERVAL ( 0 , 3.141592 ). 200 TRAPEZOIDS

APPROXIMATION TO INTEGRAL IS ***** 1.999959 *****

G(B) - G(A) IS  1    -   -1  =  2


TYPE THE CODE? 20

WHAT IS A? 0
WHAT IS B? 10
HOW MANY SUBINTERVALS ON (A,B)? 200

INTERVAL ( 0 , 10 ). 200 TRAPEZOIDS

APPROXIMATION TO INTEGRAL IS ***** 1.825092 *****

G(B) - G(A) IS  .8390715   -  -1  =  1.839072


TYPE THE CODE? 300
```

A listing of the program /FUNDTHEO/.

```
 10 PRINT"THIS PROGRAM COMPUTES APPROXIMATIONS TO THE DEFINITE"
 12 PRINT"INTEGRAL OF THE FUNCTION Y=F(X) WHICH YOU SUPPLIED ON"
 14 PRINT"LINE 100, OVER AN INTERVAL (A,B); AND COMPARES THE"
 16 PRINT"APPROXIMATION TO G(B)-G(A), WHERE Z=G(X) IS A"
 18 PRINT"PRIMITIVE OF F(X), WHICH YOU SUPPLIED IN LINE 200."
 20 PRINT
 22 PRINT "WHAT IS A";
 23 INPUT A
 24 PRINT "WHAT IS B";
 25 INPUT B
 26 IF A>=B GOTO 296
 30 PRINT "HOW MANY SUBINTERVALS ON (A,B)";
 31 INPUT N
 32 LET N=INT(ABS(N))
 40 LET S=0
 50 LET W=(B-A)/N
 60 FOR X=A TO B STEP W
100 LET Y=SIN(X)
105 IF X=B GOTO 125
110 IF X=A GOTO 125
115 LET S=S+(2*Y)
120 GOTO 130
125 LET S=S+Y
130 NEXT X
135 LET T=((B-A)/(2*N))*S
138 PRINT
140 PRINT "INTERVAL (";A;",";B;").";N;"TRAPEZOIDS"
142 PRINT
144 PRINT"APPROXIMATION TO INTEGRAL IS *****";T;"*****"
146 PRINT
195 LET L=0
197 LET X=A
200 LET Z=-1*COS(X)
205 IF L=1 GOTO 250
210 LET L=L+1
215 LET C=Z
220 LET X=B
225 GOTO 200
250 LET D=Z
255 LET R=D-C
260 PRINT "G(B) - G(A) IS ";D;"  -  ";C;" = ";R
265 PRINT
270 PRINT
275 PRINT"TYPE THE CODE";
276 INPUT E
280 IF E=20 GOTO 20
285 IF E=30 GOTO 30
290 IF E=300 GOTO 300
292 PRINT "THE CODE MUST BE 20,30, OR 300."
294 GOTO 275
296 PRINT "A MUST BE LESS THAN B."
298 GOTO 22
300 END
```

VII. SKETCHING GRAPHS OF FUNCTIONS

Constructing the Graph of a Function

In this section we are concerned with methods for sketching graphs of functions. One approach to sketching a graph of the function $y = f(x)$ is to select various values for x, to calculate the corresponding values for y by substituting values of x into the function $f(x)$, and to plot the set of ordered pairs of numbers $\{(x, f(x))\}$ on rectangular coordinate paper, using an appropriate scale for the x and y axes. When enough ordered pairs have been calculated and plotted to enable you to predict the shape of the graph with reasonable accuracy, the points can be connected by smooth lines (not necessarily straight lines) to obtain a graph of the function $y = f(x)$. If the graph has been precisely constructed, it is possible to obtain much information about the function by studying the graph.

When analyzing a function, one should consider the behavior of the function relative to any open or closed interval in the domain of the function with respect to the following mathematical concepts:

(1) Is the function increasing or decreasing?
(2) Is the function one-to-one?
(3) Is the function onto?
(4) Does the function have an inverse function?
(5) Does the function have any real zeros (roots)?
(6) Is the function strictly increasing or strictly decreasing?
(7) Is the function constant?
(8) Does the function have maximum or minimum values?
(9) Does the function have relative maximum or relative minimum values?
(10) Is the function concave upward or concave downward?
(11) Are there any real numbers for which the function is undefined?
(12) Are there any real numbers for which the function is discontinuous?

(13) Are there any real numbers for which the function does not have a derivative?
(14) What is the range of the function?
(15) What is the domain of the function?

Check your sources of information (books, other students, teachers, etc.) to make certain that you understand each mathematical term that is underlined.

Problem 24: Write computer programs which will accept as input numerical values of x and give as output values of f(x) for each of the following functions. Use your program to obtain enough points on the graph of each function so that you can plot an accurate graph of the function.

A. $f(x) = x^2-3x+2$

B. $f(x) = -x^2+3x-2$

C. $f(x) = \cos(x)$

D. $f(x) = 2\cos(x)$

E. $f(x) = \cos(2x)$

F. $f(x) = \log_4 x$

G. $f(x) = \begin{cases} \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x=0 \end{cases}$

H. $f(x) = x(\sin(x))$

I. $f(x) = \frac{e^x}{x}$

This module is a guessing game. The program called /GRAPH1/ contains five different functions of x. For any of the functions and for any real value of x which you select, the program will give as output:

(1) that value of x,
(2) the value of the function,
(3) the value of the derivative of the function,
(4) the value of the second derivative of the function.

Your job is to select as many of these functions as you want, sketch graphs of the functions and their first and second derivative functions, and try to guess what the functions are and what the derivative functions are. To sketch the graphs you will have to make shrewd selections for your values of x in order to get points which will assist you in obtaining accurate graphs.

You should try to find maximum and minimum points on the graphs, points where the functions are discontinuous or undefined, intervals where the functions are either increasing or decreasing, intervals where the functions are either concave upward or downward and points where the graphs cross the x and y axes.

After you have loaded the program /GRAPH1/ and typed:

```
RUN
```

you will get the message

```
NOW TYPE THE CODE NUMBER OF THE FUNCTION
THAT YOU WANT TO WORK WITH.
?
```

The code numbers of the functions are 100, 200, 300, 400, and 500; so you can select a function by typing any one of these numbers following the question mark. After you have selected a function you will be asked to type a value of x. When you supply the value of x you will obtain as output that value of x, together with the values of the function and its first and second derivatives. The message:

TYPE A VALUE FOR X

will be repeated five times, then you will have the option of getting values of the function and its derivatives for five more values of x or of going on to another function.

A sample output from this program is shown on the next page.

A sample run of the program /GRAPH1/.

```
NOW TYPE THE CODE NUMBER OF THE FUNCTION
THAT YOU WANT TO WORK WITH.
? 100
TYPE A VALUE FOR X.
? -2
X = -2  Y =  4 D1 = -4  D2 =  2
TYPE A VALUE FOR X.
? -1
X = -1  Y =  1 D1 = -2  D2 =  2
TYPE A VALUE FOR X.
? 0
X =  0  Y =  0 D1 =  0  D2 =  2
TYPE A VALUE FOR X.
? 1
X =  1  Y =  1 D1 =  2  D2 =  2
TYPE A VALUE FOR X.
? 2
X =  2  Y =  4 D1 =  4  D2 =  2
IF YOU HAVE ENOUGH INFORMATION ABOUT THIS FUNCTION,
TYPE STOP; FOR MORE INFORMATION, TYPE GO.
? STOP
TO OBTAIN ACCESS TO ANOTHER FUNCTION, TYPE ITS CODE
NUMBER, 100, 200, 300, 400,OR 500. TO STOP THE
PROGRAM, HIT CTRL/C.
NOW TYPE THE CODE NUMBER OF THE FUNCTION
THAT YOU WANT TO WORK WITH.
? 400
TYPE A VALUE FOR X.
? -5
X = -5  Y =  .958924  D1 =  .283662  D2 = -.958924
TYPE A VALUE FOR X.
? -2
X = -2  Y = -.909298  D1 = -.416147  D2 =  .909298
TYPE A VALUE FOR X.
? 0
X =  0  Y =  0  D1 =  1  D2 =  0
TYPE A VALUE FOR X.
? 1
X =  1  Y =  .841471  D1 =  .540302  D2 = -.841471
TYPE A VALUE FOR X.
? 10
X =  10  Y = -.544021  D1 = -.839072  D2 =  .544021
IF YOU HAVE ENOUGH INFORMATION ABOUT THIS FUNCTION,
TYPE STOP; IF YOU WANT MORE INFORMATION, TYPE GO.
? STOP
TO OBTAIN ACCESS TO ANOTHER FUNCTION, TYPE ITS CODE
NUMBER, 100, 200, 300, 400,OR 500. TO STOP THE
PROGRAM, HIT CTRL/C.
NOW TYPE THE CODE NUMBER OF THE FUNCTION
THAT YOU WANT TO WORK WITH.
? ↑C
```

A listing of the program /GRAPH1/.

```
34 PRINT " "
36 PRINT"NOW TYPE THE CODE NUMBER OF THE FUNCTION"
38 PRINT "THAT YOU WANT TO WORK WITH."
44 INPUT C
46 IF C= 100 GOTO 100
48 IF C= 200 GOTO 200
50 IF C= 300 GOTO 300
52 IF C= 400 GOTO 400
54 IF C= 500 GOTO 500
56 PRINT "YOU MUST TYPE 100,200,300,400, OR 500."
58 PRINT "TRY AGAIN."
60 GOTO 44
99 END
100 LET N=0
102 PRINT "TYPE A VALUE FOR X."
105 INPUT X
107 LET Y=X*X
110 LET D1=2*X
115 LET D2=2
120 PRINT "X = ";X;" Y = ";Y;"D1 = ";D1;" D2 = ";D2
125 LET N=N+1
130 IF N=5 GOTO 150
135 GOTO 102
150 PRINT "IF YOU HAVE ENOUGH INFORMATION ABOUT THIS FUNCTION,"
155 PRINT"TYPE STOP; FOR MORE INFORMATION, TYPE GO."
165 INPUT T$
170 IF T$="GO" GOTO 100
175 IF T$="STOP" GOTO 600
180 PRINT "YOU MUST TYPE STOP OR GO. TRY AGAIN."
185 GOTO 165
200 LET N=0
202 PRINT "TYPE A VALUE FOR X."
205 INPUT X
207 LET Y=X*(X-4)
210 LET D1=2*(X-2)
215 LET D2=2
220 PRINT "X = ";X;" Y = ";Y;" D1 = ";D1;" D2 = ";D2
225 LET N=N+1
230 IF N=4 GOTO 250
235 GOTO 202
250 PRINT "IF YOU HAVE ENOUGH INFORMATION ABOUT THIS FUNCTION,"
255 PRINT "TYPE STOP. IF YOU WANT MORE INFORMATION, TYPE GO."
260 INPUT  T$
270 IF T$="GO" GOTO 200
275 IF T$="STOP" GOTO 600
280 PRINT "YOU MUST TYPE GO OR STOP. TRY AGAIN."
285 GOTO 260
300 LET N=0
302 PRINT "TYPE A VALUE FOR X."
305 INPUT X
307 LET Y=EXP(X)
310 LET D1=Y
314 LET D2=Y
320 PRINT "X = ";X;" Y = ";Y;" D1 = ";D1;" D2 = ";D2
```

A listing of the program /GRAPH1/ (continued)

```
325 LET N=N+1
330 IF N=5 GOTO 350
335 GOTO 302
350 PRINT "IF YOU HAVE ENOUGH INFORMATION ABOUT THIS FUNCTION,"
355 PRINT"TYPE STOP; IF YOU WANT MORE INFORMATION, TYPE GO."
360 INPUT T$
370 IF T$="GO" GOTO 300
375 IF T$="STOP" GOTO 600
380 PRINT "YOU MUST TYPE GO OR STOP. TRY AGAIN."
385 GOTO 360
400 LET N=0
402 PRINT "TYPE A VALUE FOR X."
405 INPUT X
407 LET Y = SIN(X)
410 LET D1=COS(X)
415 LET D2 = -1*SIN(X)
420 PRINT "X = ";X;" Y = ";Y;" D1 = ";D1;" D2 = ";D2
425 LET N=N+1
430 IF N=5 GOTO 450
435 GOTO 402
450 PRINT "IF YOU HAVE ENOUGH INFORMATINN ABOUT THIS FUNCTION,"
455 PRINT "TYPE STOP; IF YOU WANT MORE INFORMATION, TYPE GO."
460 INPUT T$
470 IF T$="GO" GOTO 400
475 IF T$="STOP" GOTO 600
480 PRINT "YOU MUST TYPE STOP OR GO. TRY AGAIN."
485 GOTO 460
500 LET N=0
502 PRINT "TYPE A VALUE FOR X."
505 INPUT X
507 LET Y=X*(X*X-X+1)
510 LET D1=X*(3*X-2)+1
515 LET D2=6*X-2
520 PRINT "X = ";X;" Y = ";Y;" D1 = ";D1;" D2 = ";D2
525 LET N=N+1
530 IF N=5 GOTO 550
535 GOTO 502
550 PRINT "IF YOU HAVE ENOUGH INFORMATION ABOUT THIS FUNCTION,"
555 PRINT "TYPE STOP; IF YOU WANT MORE INFORMATION, TYPE GO."
560 INPUT T$
570 IF T$="GO" GOTO 500
575 IF T$="STOP" GOTO 600
580 PRINT"YOU MUST TYPE STOP OR GO. TRY AGAIN."
0585 GOTO 560
600 PRINT "TO OBTAIN ACCESS TO ANOTHER FUNCTION, TYPE ITS CODE"
605 PRINT "NUMBER, 100, 200, 300, 400,OR 500. TO STOP THE"
610 PRINT"PROGRAM, HIT CTRL/C."
650 GOTO 36
651 END
```

This module called /GRAPH2/ is a modification of /GRAPH1/. It is a game for guessing functions; however your guesses must be based upon less information than is given as output from /GRAPH1/. The program /GRAPH2/ contains the first and second derivative functions for six different functions of x. For any of these six functions and for any real value of x which you select, the program will give you for output:

(1) the value of x which you selected,
(2) the value of the derivative of the function,
(3) the value of the second derivative of the function.

Your problem is to select some or all of the six functions, to get as much information about the first and second derivatives of the functions as you need in order to sketch accurate graphs of the derivative functions, and finally to guess the functions whose first and second derivative functions are stored in the computer.

You should review the information concerning the relationship of a function to its derivative function, and be especially concerned with maximum and minimum points, inflection points, critical points, and intervals where the function is increasing or decreasing and concave upward or downward.

After you have loaded /GRAPH2/ and typed:

```
RUN
```

you will get the message

```
NOW SELECT A FUNCTION BY TYPING ITS CODE
NUMBER; 100, 200, 300, 400, 500, OR 600.
?
```

You can now select any of the six functions by typing its code number following the question mark. After you have chosen a function to study, you will be asked to type a value for x. After typing a value for x you will be given as output that value of x together with corresponding values of the first and second derivative functions of the function which you are attempting to guess.

The message

TYPE THE VALUE FOR X.

will be repeated five times, then you will have the option of getting values of the derivatives for five more values of x or of going on to another function. When you have enough information about the functions, hit the escape key.

A sample output from this program is shown on the next page.

A sample output of the program /GRAPH2/.

```
NOW SELECT A FUNCTION BY TYPING ITS CODE
NUMBER; 100,200,300,400,500, OR 600.
? 200
TYPE THE VALUE OF X.
? -1
X = -1   F1 =   .367879   F2 =   .367879
TYPE THE VALUE OF X.
? 0
X =  0   F1 =  1   F2 =  1
TYPE THE VALUE OF X.
? 2
X =  2   F1 =  7.38906   F2 =  7.38906
TYPE THE VALUE OF X.
? 3
X =  3   F1 =  20.0855   F2 =  20.0855
TYPE THE VALUE OF X.
? -2
X = -2   F1 =   .135335   F2 =   .135335
TO ASSIGN 5 MORE VALUES TO X, TYPE GO.
TO LEAVE THIS FUNCTION, TYPE STOP.
? GO
TYPE THE VALUE OF X.
? 1.5
X =  1.5   F1 =  4.48169   F2 =  4.48169
TYPE THE VALUE OF X.
? 2
X =  2   F1 =  7.38906   F2 =  7.38906
TYPE THE VALUE OF X.
? 8
X =  8   F1 =  2980.96   F2 =  2980.96
TYPE THE VALUE OF X.
? -9
X = -9   F1 =  .12341E-3   F2 =  .12341E-3
TYPE THE VALUE OF X.
? 7
X =  7   F1 =  1096.63   F2 =  1096.63
TO ASSIGN 5 MORE VALUES TO X, TYPE GO.
TO LEAVE THIS FUNCTION, TYPE STOP.
? STP
TYPE EITHER STOP OR GO. TRY AGAIN.
? STOP
TO GET ANOTHER FUNCTION TYPE ITS CODE NUMBER.
TO STOP THE PROGRAM HIT CTRL/C.
? 300
TYPE A VALUE FOR X.
? -1
X = -1 F1 = -1 F2 =  0
TYPE A VALUE FOR X.
? 0
X=0, F1 AND F2 DO NOT EXIST.
TYPE A VALUE FOR X.
? 3
X =  3 F1 =  1 F2 =  0
TYPE A VALUE FOR X.
? 5
X =  5 F1 =  1 F2 =  0
TYPE A VALUE FOR X.
? 8
X =  8 F1 =  1 F2 =  0
TO ASSIGN 5 MORE VALUES TO X, TYPE GO.
TO LEAVE THIS FUNCTION, TYPE STOP.
? ↑C
```

A listing of the program /GRAPH2/.

```
46 PRINT " "
48 PRINT "NOW SELECT A FUNCTION BY TYPING ITS CODE"
50 PRINT "NUMBER; 100,200,300,400,500, OR 600."
52 INPUT C
54 IF C=100 GOTO 100
56 IF C=200 GOTO 200
58 IF C=300 GOTO 300
60 IF C=400 GOTO 400
62 IF C=500 GOTO 500
64 IF C=600 GOTO 600
66 PRINT "YOU MUST TYPE EITHER 100,200,300,400,"
68 PRINT "500, OR 600. TRY AGAIN"
70 GO TO 52
100 LET N=0
105 PRINT "NOW TYPE A VALUE FOR X."
110 INPUT X
115 LET F1=2
120 LET F2=0
125 PRINT "X = ";X;"F1 = ";F1;" F2 = ";F2
130 LET N=N+1
135 IF N=5 GOTO 150
140 GOTO 105
150 PRINT "TO ASSIGN 5 MORE VALUES TO X, TYPE GO."
155 PRINT "TO GET ANOTHER FUNCTION OF X, TYPE STOP."
160 INPUT T$
165 IF T$="GO" GOTO 100
170 IF T$="STOP" GOTO 700
175 PRINT "YOU MUST TYPE STOP OR GO. TRY AGAIN."
180 GOTO 160
200 LET N=0
205 PRINT "TYPE THE VALUE OF X."
210 INPUT X
215 LET F1=EXP(X)
220 LET F2=F1
225 PRINT "X = ";X;" F1 = ";F1;" F2 = ";F2
230 LET N=N+1
235 IF N=5 GOTO 250
240 GOTO 205
250 PRINT "TO ASSIGN 5 MORE VALUES TO X, TYPE GO."
255 PRINT "TO LEAVE THIS FUNCTION, TYPE STOP."
260 INPUT T$
265 IF T$="GO" GOTO 200
270 IF T$="STOP" GOTO 700
275 PRINT "TYPE EITHER STOP OR GO. TRY AGAIN."
280 GOTO 260
300 LET N=0
305 PRINT "TYPE A VALUE FOR X."
310 INPUT X
315 IF X=0 GOTO 385
320 IF X<0  GOTO 326
325 IF X>0 GOTO 328
326 LET F1=-1
327 GOTO 330
328 LET F1=1
330 LET F2=0
335 PRINT "X = ";X;"F1 = ";F1;"F2 = ";F2
340 LET N=N+1
350 IF N=5 GOTO 360
```

A listing of the program /GRAPH2/ (continued)

```
355 GOTO 305
360 PRINT "TO ASSIGN 5 MORE VALUES TO X, TYPE GO."
365 PRINT "TO LEAVE THIS FUNCTION, TYPE STOP."
370 INPUT T$
372 IF T$="G0" GOTO 300
374 IF T$="STOP" GOTO 700
376 PRINT "TYPE EITHER GO OR STOP. TRY AGAIN."
380 GOTO 370
385 PRINT "X=0, F1 AND F2 DO NOT EXIST."
390 GOTO 340
400 LET N=0
405 PRINT "TYPE A VALUE FOR X."
410 INPUT X
420 LET F2=-1*COS(X)
425 PRINT "X = ";X"F1 = ";F1;"F2 = ";F2
430 LET N=N+1
435 IF N=5 GOTO 450
440 GOTO 405
450 PRINT "TO GIVE X FIVE MORE VALUES, TYPE GO."
455 PRINT "TO GET ANOTHER FUNCTION, TYPE STOP."
460 INPUT T$
465 IF T$="GO" GOTO 400
470 IF T$="STOP" GOTO 700
475 PRINT "TYPE STOP OR GO. TRY AGAIN."
480 GOTO 460
500 LET N=0
505 PRINT "TYPE A VALUE FOR X."
510 INPUT X
515 LET F1=3*X*X
520 LET F2=6*X
525 PRINT "X = ";X;"F1 = ";F1;"F2 = ";F2
530 LET N=N+1
535 IF N=5 GOTO 550
540 GOTO 505
550 PRINT "TO GIVE X MORE VALUES, TYPE GO."
555 PRINT "TO GO TO A DIFFERENT FUNCTION, TYPE STOP."
560 INPUT T$
565 IF T$="GO" GOTO 500
570 IF T$="STOP" GOTO 700
575 PRINT "TYPE EITHER STOP OR GO. TRY AGAIN."
580 GOTO 560
600 LET N=0
605 PRINT "TYPE A VALUE FOR X."
610 INPUT X
615 LET F1=X*(X-3)+2
620 LET F2=2*X-3
625 PRINT "X = ";X;"F1 = ";F1;"F2 = ";F2
630 LET N=N+1
635 IF N=5 GOTO 650
640 GOTO 605
650 PRINT "TO GIVE MORE VALUES TO X, TYPE GO."
655 PRINT "TO GO TO ANOTHER FUNCTION, TYPE STOP."
660 INPUT T$
665 IF T$="GO" GOTO 600
670 IF T$="STOP" GOTO 700
675 PRINT "YOU MUST TYPE GO OR STOP. TRY AGAIN."
680 GOTO660
700 PRINT "TO GET ANOTHER FUNCTION TYPE ITS CODE NUMBER."
705 PRINT "TO STOP THE PROGRAM HIT CTRL/C."
710 GOTO 52
715 END
```

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